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EXPERIMENTAL AND ANALYTICAL STUDY OF TRANSIENT HEAT TRANSFER FOR TURBULENT FLOW IN A CIRCULAR TUBE

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Abstract—A circular tube cooled by constant turbulent flow of water was heated stepwise with time. Variation of heat-transfer coefficient was obtained. When the tube was heated prior to the step increase of heat input, a maximum appeared in the variation of heat-transfer coefficient. Reason for the maximum is discussed. Numerical analysis is made for the same configuration as that of the experiment. The numerical results agree well with the experimental ones. An analytical expression for the variation of heat-transfer coefficient is obtained. Time required for the heat-transfer coefficient to reach steady state is studied.

NOMENCLATURE

a, thermal diffusivity [m²/s];
b, constant in equation (14);
c_p, specific heat [J/kg K];
D, inner diameter of tube [m];
d_w, thickness of tube wall [m];
g, gravity acceleration [m/s²];
g_c, conversion factor [kg m/N s²];
H, wall heat capacity per unit heat-transfer area [J/m² K];
K_T, non-dimensional wall temperature increase rate, equation (1);
k, correction factor, equations (19), (20);
Nu, Nusselt number $= \alpha D/\lambda_f$;
P, pressure [N/m²];
Pr, Prandtl number $= \nu/a_f$;
Q, $= q_n/q_{G,1}$;
q_G, heat generation rate per unit heat-transfer area [W/m²];
q_n, net heat flux to fluid [W/m²];
Re, Reynolds number, $= \bar{u}D/\nu$;
r, radius [m];
T, temperature [K];
$\bar{T}$, mean temperature [K];
t, time [s];
u, velocity [m/s];
$\bar{u}$, mean velocity [m/s];
u^*, friction velocity, $= (g_c\tau/\rho_f)^{1/2}$ [m/s];
x, axial distance [m];
y, distance from wall [m];
y^+, $= yu^*/\nu$;
Z, non-dimensional time, equation (15).

Greek symbols

α, heat-transfer coefficient [W/m² K];
β, non-dimensional wall heat capacity, equation (16);
ε_H, thermal eddy diffusivity [m²/s];
ε_M, momentum eddy diffusivity [m²/s];
Θ, $= T_w/T_{w,1}$;
λ, thermal conductivity [W/m K];
ν, kinematic viscosity of fluid [m²/s];
ρ, density [kg/m³];
τ, wall shearing stress [N/m²].

Subscripts

f, fluid;
h, heating wall;
in, inlet;
st, steady state;
tran, transient state;
w, heat-transfer wall surface;
0, initial state;
1, final state.

1. INTRODUCTION

TRANSIENT convective heat transfer is important in dynamic behaviour of a nuclear reactor or machinery. In most dynamic analyses at present, however, the heat-transfer coefficient is assumed constant during the transient; that is, the quasi-static assumption is made. The purpose of the present study is to examine transient variation of the heat-transfer coefficient analytically and experimentally.

Soliman [1] made an experiment of transient heat transfer for a turbulent flow over a flat plate with time dependent heat source. Koshkin *et al.* [2] made an experiment for a turbulent air flow in a circular tube. The heat input and/or air flow rate were varied. In their analysis, the heat-transfer coefficient at a certain moment was postulated to be determined by the first time derivatives of the wall temperature and the flow rate. For a step increase of heat input at a constant flow rate, the critical parameter derived was

$$K_T = \frac{\partial T_w}{\partial t}\cdot\frac{D^2}{[(T_w-\bar{T}_f)_1-(T_w-\bar{T}_f)_0]\cdot a_f}. \quad (1)$$

Their experimental data were correlated with the parameter by the following formula.

$$\frac{\alpha}{\alpha_{st}} = 1+\left[2.12-1.12\frac{T_w}{\overline{T}_f}\right]$$
$$[\exp(1.91\times10^{-2}K_T-2.43\times10^{-4}K_T^2)-1]. \quad (2)$$

The present author [3] made a numerical analysis of the transient heat transfer for a turbulent flow in an annulus. He obtained also an analytical solution for heat-transfer coefficient by solving a simplified energy equation for turbulent flow.

In the present paper, an experiment is described in which a circular tube cooled by water was heated stepwise with time. The flow is turbulent and steady. The experimental results are compared with the numerical results and with the analytical solution obtained in [3].

2. EXPERIMENT

2.1. *Apparatus and methods*

Figure 1 shows the experimental apparatus. A stainless steel tube cooled by water was installed horizontally. Two tubes were tested; they had different diameter and wall thicknesses as shown in Fig. 1. Both tubes were 2 m long. A heated section was 30 D located near the exit of the test section. It was heated by electric current. The 23 D of the heated section was devoted to a thermally developing section. A temperature measuring section was 5 D located in the immediate down stream of the thermally developing section.

The upstream of the heated section acted as a hydrodynamically developing region. Its length was 32 D in the test section A and 77 D in B.

Referring to the analysis by Sparrow *et al.* [4], the length at which the local heat-transfer coefficient approaches to 5% of its fully developed value is about 5 D or less. Thus, the temperature measuring section in the present experiment lay in the thermally developed region.

Variation of mean wall temperature of the measuring section $\overline{T}_h$ was obtained from an increment of the electric resistance of the tube wall. The axial temperature rise of the measuring section was less than 1% of the difference between the wall temperature and the fluid mean temperature in the steady state. In the transient state, the axial temperature rise was smaller than that in the steady state in case of the uniform axial heat input. Thus, the axial temperature distribution was neglected in the measuring section.

A resistance double bridge shown in Fig. 1 was devised to measure the resistance change of the test tube. A shunt was so designed that its resistance change was less than 1% of the resistance change of the tube during the experiment. The differential voltage ΔV was generated by increase of the tube resistance due to temperature rise. The differential voltage was recorded by an oscillograph, and then the tube wall temperature was obtained from the record.

The temperature coefficient of the resistance was measured prior to the experiment. The variation of the resistance was found linear in the range of 0–90°C:

$$\frac{R(T)}{R(0)} = \begin{cases} 1+1.18\times10^{-3}T \text{ (Test section A)} \\ 1+1.16\times10^{-3}T \text{ (Test section B)}, \end{cases} \quad (3)$$

where T is the temperature in °C and $R(T)$ and $R(0)$ are the resistances at T°C and 0°C, respectively.

Heat generation rate per unit heat-transfer area q_G was obtained from the electric current and voltage drop across the measuring section. The net heat flux from the tube wall to the fluid q_n was calculated from the heat balance relation:

$$q_n = q_G - H(\mathrm{d}\overline{T}_h/\mathrm{d}t), \quad (4)$$

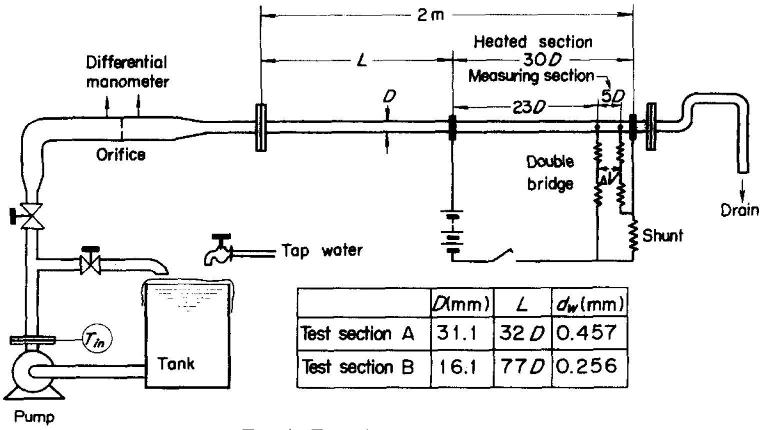
where H is the wall heat capacity per unit heat-transfer area.

The surface temperature T_w was obtained from the calculation of the radial temperature distribution in the tube wall. The transient conduction equation in the tube wall was solved numerically using the heat generation rate q_G and the surface heat flux q_n.

The heat-transfer coefficient α is defined as

$$\alpha = q_n/(T_w - \overline{T}_f), \quad (5)$$

where $\overline{T}_f$ is the mixed mean fluid temperature. It was calculated with the assumption that q_n was uniform



	D(mm)	L	d_w(mm)
Test section A	31.1	32 D	0.457
Test section B	16.1	77 D	0.256

FIG. 1. Experimental apparatus.

axially. This assumption holds strictly in the steady state and approximately in the transient state. As $T_w \gg \bar{T}_f$ in both steady and transient states, a slight error in $\bar{T}_f$ did not affect α so much.

The test tube was installed horizontally in the air without any thermal insulation over its outer surface. The heat lost from the outer surface was estimated, referring to Hellums–Churchill's [5] and Gebhart's [6] analyses for the transient natural convection. The heat lost was found less than 1% of the heat transferred to the water in both steady and transient states.

The heat generation rate was increased stepwise at $t = 0$. Voltage applied was so adjusted that the temperature difference between wall and fluid became roughly 10 K in the final steady state. The heating current was about 200–400 A for the test section A and 150–300 A for the test section B. In the experiment, the power input $q_{G,1}$ decreased slightly with time owing to increase of the internal resistance of batteries. The initial heat input $q_{G,0}$ was zero or not zero. The effect of the initial heat input on the transient heat transfer was studied.

2.2. *Experimental results*

The steady-state heat-transfer coefficient obtained from the experiment agreed well with the correlation for a circular tube in the steady state: $Nu = 0.023\,Re^{0.8}\,Pr^{0.4}$.

Some results of the transient experiments are shown in Fig. 2 together with numerical and quasi-static solutions. Two examples for different Reynolds numbers are compared in the figure. The numerical and quasi-static solutions are obtained for the same Reynolds number as that of the experiment.

The heat-transfer coefficient decreases with time and reaches the steady-state value asymptotically. The time required for the heat-transfer coefficient to reach the steady state becomes small when the Reynolds number is large.

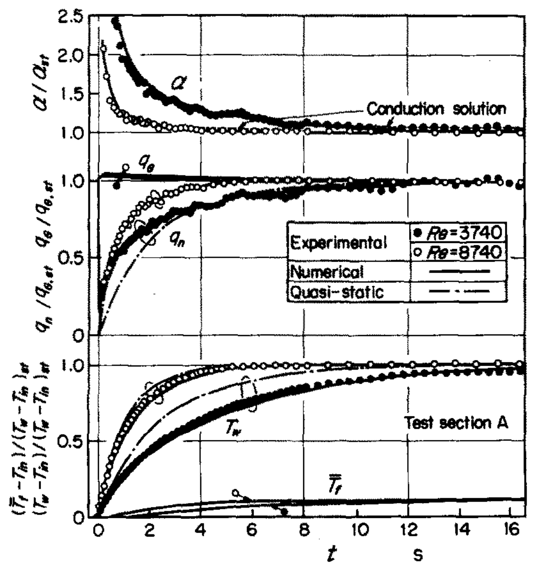


FIG. 2. Experimental results compared with numerical and quasi-static solutions.

The quasi-static solution in Fig. 2 is the solution with the quasi-static assumption; that is, the heat-transfer coefficient is assumed equal to its steady state value. The difference between the quasi-static solution and the experimental result increases with decreasing Reynolds number.

The numerical solution and the conduction solution illustrated in Fig. 2 will be explained in the following chapter.

3. ANALYSIS

3.1. *Numerical analysis*

The same problem as the experiment is analysed numerically. Assumptions are as follows: (1) the turbulent flow is fully developed and does not change with time, (2) physical properties are independent of the temperature, (3) the outer surface of the tube is insulated, (4) the turbulent eddy diffusivities, ε_M and ε_H, do not change with time.

The last assumption will be examined in some detail. Because the flow is steady and its physical properties are assumed independent of the temperature, the momentum eddy diffusivity ε_M is constant. The eddy diffusivity ratio $\varepsilon_H/\varepsilon_M$ may change in the transient state. However, an order of time of this change is that of the heat exchange between turbulent eddies, and is much smaller than that of the wall temperature variation. Thus, the thermal eddy diffusivity can also be assumed unchanging with time.

The momentum equation for the fluid is

$$\frac{g_c}{\rho_f}\frac{\partial P}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[(\varepsilon_M+\nu)r\frac{\partial u}{\partial r}\right], \tag{6}$$

and the energy equation is

$$\frac{\partial T_f}{\partial t} + u\frac{\partial T_f}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[(\varepsilon_H+a_f)r\frac{\partial T_f}{\partial r}\right], \tag{7}$$

where r and x are the co-ordinates in radial and axial directions, respectively. The heat-conduction equation in the tube wall is

$$\frac{\partial T_h}{\partial t} = a_w\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_h}{\partial r}\right) + \frac{q_G}{H}. \tag{8}$$

Boundary conditions are $T_f = T_{in}$ at the starting point of the heating section ($x = 0$); $\partial u/\partial r = 0$ and $\partial T_f/\partial r = 0$ on the tube axis ($r = 0$); $\partial T_h/\partial r = 0$ on the outer surface ($r = r_0$); $u = 0$, $T_f = T_h$ and $\lambda_f(\partial T_f/\partial r) = \lambda_w(\partial T_h/\partial r)$ on the heat-transfer surface ($r = r_w$). The initial steady state for $q_{G,0}$ is first calculated, and then the transient state.

The momentum eddy diffusivity used is the Reichardt's [7] correlation multiplied by the damping factor postulated by Wilson–Medwell [8].

$$\frac{\varepsilon_M}{\nu} = 0.4\frac{r_w u^*}{3\nu}\left[0.5+\left(\frac{r}{r_w}\right)^2\right]\cdot\left[1-\left(\frac{r}{r_w}\right)^2\right][1-\exp(-y^+/A^+)]. \tag{9}$$

The damping constant A^+ is so decided that the steady state heat-transfer coefficient obtained numerically coincides with that obtained experimentally. The value of A^+ is found about 40 for $Re \geqslant 10^4$ and larger for $Re < 10^4$.

The eddy diffusivity ratio $\sigma = \varepsilon_H/\varepsilon_M$ is given referring to Mizushina [9] as follows:

$$\sigma = 1.5\phi\{1-\exp(-1/\phi)\} \tag{10}$$

$$\phi = (\varepsilon_M/\nu)Pr/[4.13+0.743(\varepsilon_M/\nu)^{1/2}Pr^{1/3}]. \tag{11}$$

In the viscous sublayer, $\varepsilon_H = \varepsilon_M$ is assumed.

Numerical results are compared with the experimental ones in Fig. 2. The agreement is good.

The conduction solution in Fig. 2 is a numerical solution of the equation

$$\frac{\partial T_f}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[(\varepsilon_H+a_f)r\frac{\partial T_f}{\partial r}\right]. \tag{12}$$

This is derived from equation (7) by neglecting the convection term $u(\partial T_f/\partial x)$. Equation (12) has the same form as the thermal conduction equation; so, its solution is called the conduction solution in the present paper.

The heat-transfer coefficient by the conduction solution agrees well with the experimental one except at large times. The reason why the convection term can be neglected at small times was discussed in [3]. Briefly, it is because $\partial T_f/\partial x$ remains zero at small times if the heat input is initially zero and axially uniform.

3.2. *Simplified analytical solution*

If the transient variation of heat-transfer coefficient is known, the variation of wall temperature can be obtained by solving the one-dimensional energy equation instead of solving the two-dimensional one, equation (7). The one-dimensional energy equation can be written as

$$\frac{\partial \bar{T}_f}{\partial t} + \bar{u}\frac{\partial \bar{T}_f}{\partial x} = \frac{2}{r_w(\rho c_p)_f}\alpha(x,t)(T_w-\bar{T}_f). \tag{13}$$

If α in equation (13) is assumed to be constant with time, the quasi-static solution is then obtained.

An approximate solution for the transient variation of the heat-transfer coefficient was derived in [3], by solving the conduction equation, equation (12), with the approximation:

$$\varepsilon_H + a_f = a_f(by+1)^n. \tag{14}$$

Here n in equation (14) was assumed 2, and then equation (12) was solved analytically.

It was found [3] that the transient variation of the heat-transfer coefficient was determined by two non-dimensional parameters; i.e. a non-dimensional time

$$Z = \frac{\alpha_{st}^2}{4(\lambda\rho c_p)_f}t, \tag{15}$$

and a non-dimensional wall heat capacity

$$\beta = \frac{\alpha_{st}H}{(\lambda\rho c_p)_f}. \tag{16}$$

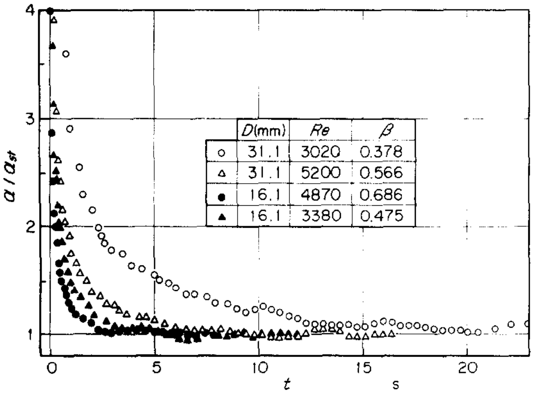


FIG. 3(a). Variation of heat-transfer coefficient plotted vs real time in s.

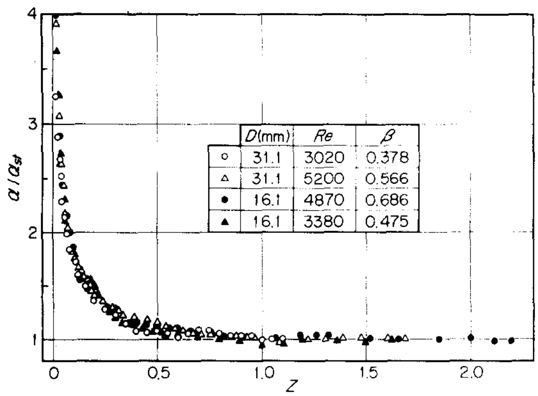


FIG. 3(b). Variation of heat-transfer coefficient plotted vs non-dimensional time Z.

Figure 3(a) is variation of the heat-transfer coefficient with the real time in second for the case of $q_{G,0} = 0$. The variation depends on the Reynolds number, the tube diameter and wall thickness. The same data are replotted against the nondimensional time Z in Fig. 3(b). The variation of the heat-transfer coefficient is well correlated by Z and slightly depends on β. It is thus indicated that the non-dimensional time Z is a critical parameter relating with the variation of heat-transfer coefficient.

With Z and β, the variation of heat-transfer coefficient is expressed as [3]:

$$\alpha_{tran}(Z,\beta) = \alpha_{st}\frac{Q(Z,\beta)}{\Theta(Z,\beta)}, \tag{17}$$

where Q and Θ are the non-dimensional heat flux and wall temperature, respectively. Their analytical forms are given in Table 1.

In substitution of equation (17) into equation (13), equation (13) is solved taking the time variation of α into consideration. This solution will be called the "simplified solution". The axial variation of α is assumed as follows:

$$\alpha(x,t) = \begin{cases}\alpha_{tran}(t), & \text{if} \quad \alpha_{tran}(t) \geqslant \alpha_{st}(x)\\ \alpha_{st}(x), & \text{if} \quad \alpha_{tran}(t) < \alpha_{st}(x),\end{cases} \tag{18}$$

where $\alpha_{st}(x)$ is the axial distribution of heat-transfer coefficient in the steady state. It is obtained from the

Table 1. Θ and Q

$$Z' = [(2/\beta)-1]^2 Z$$

$\beta \neq 1, \beta \neq 0$

$$\Theta = \frac{1+(1-\beta)^2}{2(1-\beta)^2}\operatorname{erf}\sqrt{Z} + \frac{\beta(\beta-2)}{2(1-\beta)^2}(1-e^{-Z}e^{Z'}\operatorname{erfc}\sqrt{Z'}) + \frac{1}{(1-\beta)}\frac{2}{\sqrt{\pi}}\sqrt{Z}\,e^{-Z}(1-\sqrt{\pi}\sqrt{Z}\,e^{Z}\operatorname{erfc}\sqrt{Z})$$

$$Q = \frac{2-\beta}{2(1-\beta)}(1-e^{-Z}e^{Z'}.\operatorname{erfc}\sqrt{Z'}) - \frac{\beta}{2(1-\beta)}\operatorname{erf}\sqrt{Z}$$

$\beta = 1$

$$\Theta = \operatorname{erf}\sqrt{Z} + \frac{2}{\sqrt{\pi}}\sqrt{Z}\,e^{-Z} - \frac{4}{\sqrt{\pi}}\sqrt{Z}\,e^{-Z}(1-\sqrt{\pi}\sqrt{Z}\,e^{Z}\operatorname{erfc}\sqrt{Z})\cdot(1+Z)$$

$$Q = \operatorname{erf}\sqrt{Z} + \frac{2}{\sqrt{\pi}}\cdot\sqrt{Z}\,e^{-Z}(1-\sqrt{\pi}\sqrt{Z}\,e^{Z}\operatorname{erfc}\sqrt{Z})$$

$\beta = 0$

$$\Theta = \operatorname{erf}\sqrt{Z} + \frac{2}{\sqrt{\pi}}\sqrt{Z}\,e^{-Z}(1-\sqrt{\pi}\sqrt{Z}\,e^{Z}\operatorname{erfc}\sqrt{Z})$$

$$Q = 1.$$

numerical solution in the steady state; or, an approximate correlation may be used.

The simplified solution is compared with the experimental one in Fig. 4. The broken line shows the simplified solution. The simplified solution coincides with the experimental result better than the quasi-static solution; but some error still exists. The error in heat-transfer coefficient by the simplified solution increases with decrease of β, and becomes at most 30% when $\beta < 0.5$.

The error is due to the approximation of equation (14), not due to the neglection of the convection term in the energy equation. It has been already found that the convection term is negligible except at large times (see Fig. 2).

A correction is attempted empirically to get a better agreement with the experiment. A correction factor k is introduced in equation (17) as follows:

$$\alpha(Z,\beta) = \alpha_{st}\frac{Q(Z,\beta)}{\Theta(kZ,k\beta)}. \quad (19)$$

Then, good agreement with the experiment is obtained, if k is given as

$$k = \begin{cases} 1+0.6(1-\beta/4), & 0 \leqslant \beta < 4 \\ 1, & 4 \leqslant \beta. \end{cases} \quad (20)$$

The correction factor is found empirically for cooling by water. The simplified solution with this correction is shown in Fig. 4 by the solid line. Agreement with the experiment is thus improved.

Equation (19) is not rigorous as far as $k \neq 1$. When $Z \to \infty$, however, α given by equation (19) becomes α_{st}; and when $Z \to 0$, equation (19) becomes the correct form:

$$\alpha = \alpha_{st}/(\pi Z)^{1/2} = 2/\pi^{1/2}\cdot[(\lambda\rho c_p)_f/t]^{1/2}, \quad (21)$$

if $\beta \neq 0$. Equation (19) is thus the rigorous solution for both small and large times.

The "simplified solution" or "analytical solution" hereafter is the solution with the correction.

Figure 5 shows the times required for heat-transfer coefficient or wall temperature to reach the steady state for $q_{G,0} = 0$. The steady state time $Z_{st,\alpha}$ is defined as the time for α to decrease to 1.1 α_{st} and $Z_{st,w}$ is the time for $(T_w - T_f)$ to increase to 90% of the steady state value. Both the steady state times for two different test sections are well correlated by Z and β.

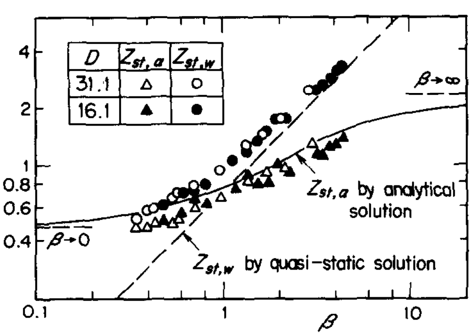


FIG. 5. Times required for heat-transfer coefficient and wall temperature to reach steady state.

The steady state time for heat-transfer coefficient $Z_{st,\alpha}$ is about $Z \sim 1$ and slightly dependent on β. Since the heat-transfer coefficient approaches the steady state asymptotically, it is often sufficient to know only the order of magnitude of the steady state time. For such purpose, the time required for heat-transfer coefficient to reach the steady state may be given as $Z \sim 1$ or

$$t_{st,\alpha} \sim 4(\lambda\rho c_p)_f/\alpha_{st}^2. \quad (22)$$

The steady state time for wall temperature $Z_{st,w}$ obtained by experiment agrees with that by the quasi-static solution, when $\beta \gg 1$. It was found [3] that, the variation of wall temperature is quasi-static if $\beta \gg 1$. In this case the transient behaviour of heat-transfer coefficient has no large effect on the wall temperature.

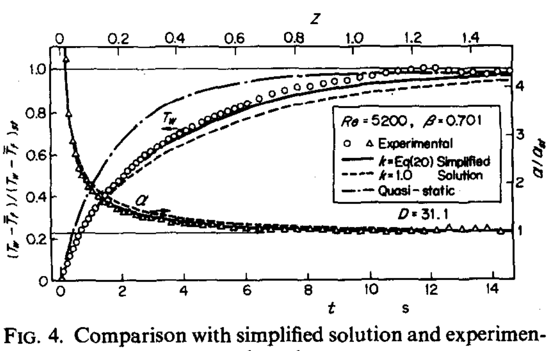


FIG. 4. Comparison with simplified solution and experimental results.

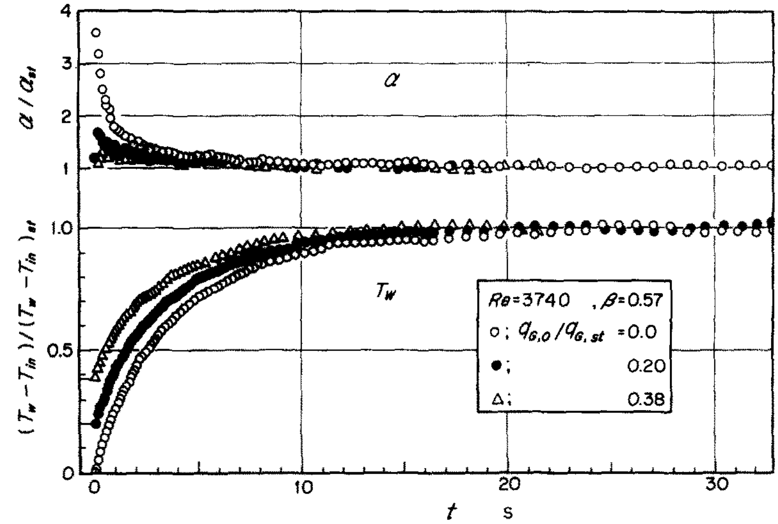


FIG. 6. Experimental results for various initial heat inputs.

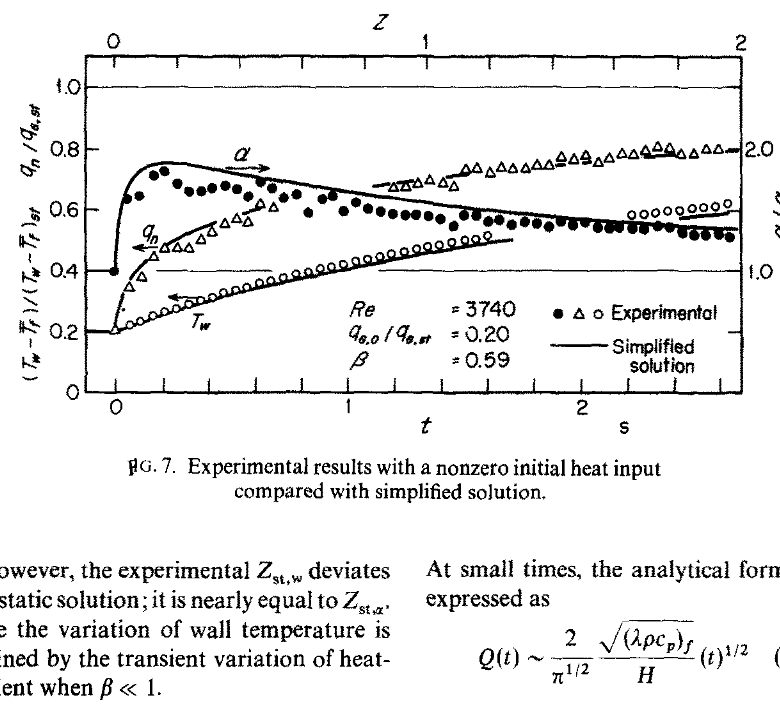


FIG. 7. Experimental results with a nonzero initial heat input compared with simplified solution.

When $\beta \ll 1$, however, the experimental $Z_{st,w}$ deviates from the quasi-static solution; it is nearly equal to $Z_{st,\alpha}$. This is because the variation of wall temperature is mainly determined by the transient variation of heat-transfer coefficient when $\beta \ll 1$.

4. EFFECT OF INITIAL POWER INPUT

Figure 6 shows the results by experiment for three different initial heat inputs. When $q_{G,0} \neq 0$, a maximum appears in the variation of the heat-transfer coefficient. The maximum decreases with increase of $q_{G,0}$.

The experimental result for $q_{G,0} \neq 0$ is compared with the simplified solution in Fig. 7. The maximum in the heat-transfer coefficient is seen clearly in the figure.

The maximum in heat-transfer coefficient is already found analytically in [3]. The general form of equation (17) inclusive of $q_{G,0} \neq 0$ is [3]

$$\alpha(t) = \alpha_{st} \cdot \frac{Q(t) + (q_{G,0}/q_{G,1}) \cdot [1 - Q(t)]}{\Theta(t) + (q_{G,0}/q_{G,1}) \cdot [1 - \Theta(t)]}. \quad (23)$$

At small times, the analytical forms of Q and Θ are expressed as

$$Q(t) \sim \frac{2}{\pi^{1/2}} \frac{\sqrt{(\lambda \rho c_p)_f}}{H} (t)^{1/2} \quad (t \to 0, H \neq 0) \quad (24)$$

$$\Theta(t) \sim \frac{\alpha_{st}}{H} t \quad (t \to 0, H \neq 0). \quad (25)$$

When $q_{G,0} = 0$, α is given by equation (21) and becomes infinite as $t \to 0$. When $q_{G,0} \neq 0$, however, α is equal to α_{st} and stays finite.

This is explained physically as follows. When $q_{G,0} = 0$, the temperature is uniform in fluid at $t = 0$; so, the heat-transfer coefficient at small times is determined by the transient heat conduction in the fluid. When $q_{G,0} \neq 0$, a steady state temperature distribution already exists at $t = 0$. The heat-transfer coefficient is determined by the temperature distribution; thus $\alpha = \alpha_{st}$ at $t \to 0$.

Koshkin *et al.* [2] assumed that the transient α/α_{st} was correlated by equation (2) using the nondimensional wall temperature increase rate K_T given

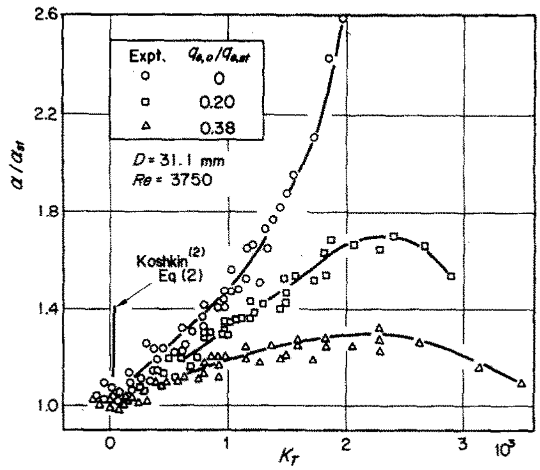


FIG. 8. Heat-transfer coefficient ratio α/α_{st} vs non-dimensional wall temperature increase rate K_T.

by equation (1). The ratio α/α_{st} obtained in the present experiment is plotted vs K_T in Fig. 8. Equation (2) is also plotted in Fig. 8 for $T_w/\bar{T}_f \sim 1$. The present experimental results do not agree with equation (2).

The present results in Fig. 8 show that α/α_{st} depends on $q_{G,0}$ and cannot be correlated only by K_T. This fact contradicts the Koshkin's assumption.

When $q_{G,0} \neq 0$, α/α_{st} is 1 and dT_w/dt is not zero at $t = 0$. So, the $\alpha/\alpha_{st} - K_T$ curve starts at where $\alpha/\alpha_{st} = 1$ and $K_T \neq 0$. As K_T decreases, the curve has a maximum and finally reaches the point of $\alpha/\alpha_{st} = 1$ and $K_T = 0$. The maximum depends on $q_{G,0}$. When $q_{G,0} = 0$, the curve starts with $\alpha/\alpha_{st} \to \infty$. Thus, the $\alpha/\alpha_{st} - K_T$ curve depends on $q_{G,0}$; so, α/α_{st} cannot be correlated by K_T alone.

Koshkin *et al.* [2] made the experiment with air. In this case, β becomes so large that the wall temperature variation is nearly quasi-static [3]. This means that high accuracy is required for the wall temperature measurement, to calculate the heat flux from variation of the wall temperature. Some factors ignored in the present study may play an important role in Koshkin's experiment. Effect of variation of the physical properties, for example, should be studied further.

5. CONCLUSIONS

(1) The steady state times for heat-transfer coefficient and for wall temperature can be correlated by the non-dimensional parameters Z and β for different tube diameters and wall thicknesses.

(2) Analytical solution of the heat-transfer coefficient agrees with the experimental one within error of 30%. If the correction factor given by equations (19) and (20) is introduced, good agreement is then obtainable for water.

(3) When the initial heat input is not zero, a maximum appears in variation of the heat transfer coefficient.

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RECHERCHE EXPERIMENTALE ET ANALYTIQUE SUR LE TRANSFERT THERMIQUE TRANSITOIRE POUR L'ECOULEMENT TURBULENT DANS UN TUBE CIRCULAIRE

Résumé—Un tube circulaire, refroidi par un écoulement turbulent constant d'eau, est réchauffé par gradins dans le temps. On obtient une variation du coefficient de transfert de la chaleur. Quand le tube est réchauffé, avant l'augmentation échelonnée du chauffage, un maximum apparait dans la variation du coefficient. On discute une cause probable de cette apparition du maximum. Une analyse numérique est faite sur la même configuration que celle de l'expérimentation. Ses résultats sont bien d'accord avec ceux expérimentaux. Une expression analytique pour la variation du coefficient de transfert est donnée. On étudie le temps nécessaire pour que le coefficient de transfert thermique atteigne la valeur stationnaire.

EXPERIMENTELLE UND ANALYTISCHE UNTERSUCHUNG DES INSTATIONÄREN WÄRMEÜBERGANGES BEI TURBULENTER STRÖMUNG IN EINEM RUNDEN ROHR

Zusammenfassung—Ein von einer turbulenten Wasserströmung gekühltes Rohr wird zeitlich stufenweise erhitzt und die Veränderung des Wärmeübergangskoeffizienten wird gemessen. Wenn das Rohr am Anfang erhitzt wird, ergibt sich ein Maximum in der Veränderung des Wärmeübergangskoeffizienten. Der Grund für das Maximum wird diskutiert. Eine numerische Analyse wird für den gleichen Aufbau wie bei dem Experiment durchgeführt. Die numerischen Ergebnisse stimmen gut mit den experimentellen Ergebnissen überein. Eine Formel für die Veränderung des Wärmeübergangskoeffizienten wird angegeben. Die erforderliche Zeit für Erreichen des stationären Zustandes des Wärmeübergangskoeffizienten wird untersucht.

ТЕОРЕТИКО-ЭКСПЕРИМЕНТАЛЬНОЕ ИССЛЕДОВАНИЕ НЕСТАЦИОНАРНОГО ТЕПЛООБМЕНА ДЛЯ ТУРБУЛЕНТНОГО ТЕЧЕНИЯ В КРУГЛОЙ ТРУБЕ

Аннотация — Анализируется процесс ступенчатого во времени нагрева круглой трубы, охлаждаемой турбулентным потоком воды. Получена зависимость коэффициента теплообмена от влияющих факторов. При нагреве трубы до начала ступенчатого изменения наблюдается максимум на кривой временного изменения коэффициента теплообмена. Обсуждается причина появления максимума. Проводится численный анализ для условий, реализованных в эксперименте. Численные результаты хорошо согласуются с измерениями. Получено аналитическое выражение для изменения коэффициента теплообмена. Приведены оценки времени, необходимого для достижения установившихся значений коэффициента теплообмена.